# MATHEMATICS HL

## Overall grade boundaries

## Higher level

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 13 | 14 - 28 | 29 - 41 | 42 - 53 | 54 - 64 | 65 - 76 | 77 - 100 |

# Higher level internal assessment

## Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 6 | 7 - 13 | 14 - 18 | 19 - 23 | 24 - 29 | 30 - 34 | 35 - 40 |

## The range and suitability of the work submitted

A few excellent portfolios were produced in this first November session of the new syllabus. The changes to the assessment criteria were significant, yet expectations seemed to be well understood by most teachers and their students. The moderators have made a number of observations that are summarised below:

### The tasks in the new syllabus

The majority of the portfolio tasks were taken from the Teacher Support Material (TSM) for Mathematics HL, but a few good teacher-designed tasks were also noted. Unfortunately, where tasks were taken from the previous TSM, the new assessment criteria were not adequately met. Unless significant modifications are made, these older tasks should not be used. Some examples of tasks deficient in content included investigations (type I) that precluded the use of technology and modelling tasks (type II) in which the model was not created by the student as required in the new criteria, but given to students within the task.

Tasks taken from the TSM for Mathematics SL are not at a suitable level for Mathematics HL and should not be used.

The “Extended Closed Problem Solving” tasks (old type II) were deleted from the new internal assessment structure. Any such tasks taken from the previous TSM and submitted in future sessions will be subject to a non-compliance penalty, as well as a significant loss of marks in criteria C and D as the new achievement levels will not be met.

### Criteria clarification and notes to moderators

The following criteria notes were directed to moderators after the standardisation meeting in April and are presented here for all teachers to consider.

### Criterion A: use of notation and terminology

Tasks will probably be set before students are aware of the notation and/or terminology to be used. Therefore the key idea behind this criterion is to assess how well the students’ use of terminology describes the context.

 

Teachers should provide an appropriate level of background knowledge in the form of notes given to students at the time the task is set.

Correct mathematical notation is required, but it can be accompanied by calculator notation, particularly when students are substantiating their use of technology.

This criterion addresses appropriate use of mathematical symbols (for example, use of “ ≈ ” instead of “ = ” and proper vector notation).

Word processing a document does not increase the level of achievement for this criterion or for criterion B.

Students should take care to write in appropriate mathematical symbols if the word processing software does not supply them. Calculator/computer notation should not be used. Notation such as *x* ^ 2 or ABS(*x*) should not be used and such use will be penalised. A single shortcoming would not preclude the awarding of level 2.

Terminology may depend on the task. In the case of Type I (Investigation) tasks, terminology may include terms devised by the candidate (e.g. “slide”, “shift”), provided that such terms reasonably reflect the appropriate mathematical concept.

**Criterion B: communication**

This criterion also assesses how coherent the work is. The work can achieve a good mark if the reader does not need to refer to the wording used to set the task. In other words, the task can be marked independently.

Level 2 cannot be achieved if the student only writes down mathematical computations without explanation.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached to the end of the document. Graphs must be correctly labelled and must be neatly drawn on graph paper. Graphs generated by a computer program or a calculator “screen dump” are acceptable providing that all items are correctly labelled, even if the labels are written in by hand. Colour keying the graphs can increase clarity of communication.

If, in reading a candidate’s work, the teacher has to pause to clarify where a result came from or how it was achieved (“WHOA! Where did that come from?!”), this generally indicates flawed communication.

Computer/calculator output may need clarification. Graphs generated by a calculator or computer should present the variables and labels appropriate to the task. Hand-written labels may need to be added to screen dumps or printouts if the software doesn’t provide for custom labels.

A single shortcoming would not preclude the awarding of level 3.

**Criterion C: mathematical process**

Type I—mathematical investigation: searching for patterns

Students can only achieve a level 3 if the amount of data generated is sufficient to warrant an analysis.

This is the process of getting the statement. Student gets 4 if everything is ready for the statement. The correctness of the statement is assessed in D.

 

If student gives a proof of the correct statement, no further cases need be investigated to award a level 5.

Type II—mathematical modelling: developing a model

At achievement level 5, applying the model to other situations could include, for example, a change of parameter or more data.
Any form of definition of variables, parameters constraints (informal/implied) is acceptable (e.g. labelling a graph or table, noting domain and range).

**Criterion D: results**

Type I—mathematical investigation: generalization

A student who gives a correct formal proof of the general statement that does not take into account scope or limitations would achieve level 4.

It is important to note the difference between "a (i.e. any) general statement" in level 2 and "the general statement" in level 3.

Type II—mathematical modelling: interpretation

"Appropriate degree of accuracy" means appropriate in the context of the task.

**Criterion E: use of technology**

The emphasis in this criterion is on the contribution of the technology to the mathematical development of the task rather than to the presentation and/or communication.

The level of calculator or computer technology varies from school to school. Therefore teachers should state the level of the technology that is available to their students. While printed output is not required, some statement confirming appropriate use of technology (from the teacher or student) is necessary.

Using a computer and/or a GDC to generate graphs or tables may not significantly contribute to the development of the task, and therefore may not merit more than a level 1.

**Criterion F: quality of work**

Students who satisfy all the requirements correctly achieve level 1. For a student to achieve level 2, work must show precision, insight and a sophisticated level of mathematical understanding.

Award level 2 only if the work presented is beyond ordinary expectations. The teacher will take a pause to admire the quality of such work ("Wow! Now, that's impressive!").

Only a totally inadequate response would receive 0.

## Candidate performance against each criterion

Candidates generally performed well against criterion A. The use of computer notation seemed to be very limited. Correct terminology should include the use of correct mathematical vocabulary, such as "substitute" instead of "plug in".

Some students produced excellent pieces of technical writing. On the other hand, others have merely shown the steps to the solutions of problems and their work was found to be severely lacking in explanation and links within and between parts of the tasks. To meet the expectations in criterion B, students should be given explicit instructions to structure and present their work.

 

In criteria C and D, students have fared well, but the assessments by their teachers have been notably lenient. In type I tasks, sufficient data have often not be generated to justify the formulation of a conjecture. Where several intermediate general statements were derived, the proof of *the* general statement was not always evident to warrant full marks. In type II tasks, the statement of variables, perhaps in a “let statement”, has often been implied, but should be explicit. Some realisation of the significance of the results obtained in terms of the model when compared to the actual situation needs to be given, but few students endeavoured to analyse their findings.

Success in meeting criterion E varied considerably. Generally, more sophisticated use of technology was noted in many portfolios, yet a few candidates demonstrated a very limited use of the GDC or of computer software in some of the type I tasks. The *resourceful* use of technology should be evident in the enhancement of the work, rather than by the mere addition of redundant graphs of limited value.

Many good pieces of work were noted; however, the awarding of full marks in criterion F requires more than completion and correctness as noted in the clarifications above.

## Recommendations for the teaching of future candidates

Teachers should select tasks that provide students with a variety of mathematical activities suitable at higher level. Tasks taken from the Mathematics SL TSM do not meet HL requirements and may be subject to a non-compliance penalty in future sessions.

Teachers are expected to write directly on their students’ work not only to provide feedback to students but information to moderators as well. Some samples contained very few teacher comments.

Original student work must be sent in the sample, as teacher comments on photocopies are often illegible. Moderation was extremely difficult when it was not possible to determine the basis upon which the teacher awarded marks.

Moderators find the background to each portfolio task very useful in determining the context in which the task was given when confirming the achievement levels awarded. This information must accompany each sample.

If a teacher-designed task is submitted, a solution key must accompany the portfolios in order that moderators can justify the accuracy of the work, and appreciate the level of sophistication demonstrated in the work.

# Higher level paper one

## Component grade boundaries

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **Mark range**: | 0 - 15 | 16 - 30 | 31 - 47 | 48 - 62 | 63 - 76 | 77 - 91 | 92 - 120 |

## The areas of the programme and examination that appeared difficult for the candidates

The topics found difficult by many candidates were probability and statistics, related rates of change and changing the base of a logarithm.

 

## The areas of the programme and examination in which candidates appeared well prepared

The level of knowledge, understanding and skill demonstrated was generally good. Many candidates proved to be competent in the use of a GDC and used their calculator appropriately. In several questions involving the taking of a square root, many candidates failed to realise that this results in a $\pm$. Many candidates suffered an accuracy penalty with some candidates incorrectly rounding throughout the paper.

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1**

Most candidates found the matrix inverse correctly using their GDC. Those who tried to invert either by row operations or cofactors usually made arithmetic errors. It was unfortunate that candidates who ignored the word **hence** in (b) gained no credit.

**Question 2**

Most candidates solved this correctly, either using the binomial theorem or successive multiplication of brackets.

**Question 3**

It was disappointing to see some candidates obtaining

$$f'(x) = \frac{1}{3x+1}$$

**Question 4**

Most candidates solved (a) correctly but the Bayesian element in (b) defeated many candidates.

**Question 5**

This caused problems for some candidates who were unable to progress beyond

$$\frac{a}{1-r} = 32 \text{ and } \frac{a(1-r^4)}{1-r} = 30$$

**Question 6**

Many candidates failed to obtain all 4 roots, usually by not realising that $\tan 2\theta$ could be -1 as well as +1. Candidates who obtained all 4 roots using a graphical method on their GDC gained full credit only if they drew an appropriate graph in their answer book.

**Question 7**

In (a), some candidates confused $P(X \leq 1)$ with $P(X = 1)$ leading to an incorrect mean. Many candidates suffered an accuracy penalty by rounding the value of the mean to 3.

**Question 8**

Part (a) was well done by many candidates, although some evaluated $100 \times 1.05^{19}$ instead of $100 \times 1.05^{20}$. Part (b) caused problems for some candidates with many unaware of the possibility of using logs.

 

**Question 9**

Many candidates found the correct probability in (a) but were unable to use it to find the required expected number. In (b), some candidates who obtained the correct answer showed no working, losing a mark in consequence.

**Question 10**

This was successfully solved by many candidates. Those who expressed $z_1$ and $z_2$ in the form $a + b\mathrm{i}$ at the beginning were generally unsuccessful and they wasted valuable time.

**Question 11**

This was well done by many candidates. Candidates who found the sine of the angle using a vector product were given full credit although this method is not recommended.

**Question 12**

Many candidates solved this problem in their heads with little or no working so arithmetic errors were not uncommon.

**Question 13**

A fairly common approach was to evaluate the distance of P from the plane $\Pi$. Having done this, they generally made no further progress.

**Question 14**

Many candidates failed to make the crucial step, namely

$$\log_x 5 = \frac{1}{\log_5 x}$$

Even candidates who used this then failed to realise (as in Question 6) that when you take a square root, you introduce $\pm$.

**Question 15**

This proved difficult for many candidates who were unable to go beyond the following equations, obtained by making the curves touch

$$2x^2 + (k-2)x + k + 4 = 0 \text{ and } 4x = 2 - k$$

Some candidates chose the resulting positive value for $k$ although this was given almost full credit using the rule for a mis-read.

**Question 16**

Many candidates realised, by using the sine rule, that sinC = 0.7 although many of these failed to realise that this gave 2 possible values for C. Some candidates used the cosine rule, which is a valid method, although many were unable to carry this through to the end. Several other ingenious methods were seen, usually giving the correct exact answer, namely $7\sqrt{51}/4$.

**Question 17**

Many attempts at this question were disappointing, with many candidates failing to realise that the variables could be separated.

**Question 18**

Many candidates failed to realise that the first thing to do was to find out where the curves met, without which little progress was possible.

 

**Question 19**

Questions on related rates of change usually cause problems for many candidates and this was no exception.

**Question 20**

Most candidates were unable to solve this problem. Only a few candidates realised that the crucial step was to note that A + B + C = $\pi$ and candidates who did spot this usually made some progress. Some candidates filled the page with applications of the cosine rule although it should have been obvious that this would lead nowhere.

## Recommendations and guidance for the teaching of future candidates

- Ensure that candidates are aware of the accuracy rules. Many candidates are losing a mark for incorrect rounding.
- Advise candidates to use their GDC to invert matrices. Candidates who use row reduction or cofactors often make arithmetic errors.
- Ensure that candidates are aware that all working must now be shown on Paper 1. Many candidates lost marks for not showing working, especially in questions involving the use of a GDC.
- Ensure that candidates clearly understand the meaning of the word 'hence'.

# Higher level paper two

## Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 16 | 17 - 32 | 33 - 46 | 47 - 60 | 61 - 73 | 74 - 87 | 88 - 120 |

## The areas of the programme and examination that appeared difficult for the candidates

Questions which many candidates had difficulty in answering were those based on the contents of Complex Numbers and certain concepts related to Differentiation.

## The areas of the programme and examination in which candidates appeared well prepared

I found that the level of knowledge, understanding and skill demonstrated in the scripts I marked this session was quite good on the whole. However, I found that many graphs in question 4 were extremely untidy and generally messy: no clear scales on axes, little attention paid to the domain defined in the question, etc. Good use of the gdc was evident in most cases. Although overall "show that…" questions seem to be well tackled, there is still quite a high proportion of students who verify that the answer given does in fact satisfy the identity / equality in question. Lack of time was not an issue as far as I could tell – most scripts were complete and candidates seemed to have worked on all the questions for a fair amount of time. There were a certain percentage of scripts from students who did not manage to score more than 10 or 15 marks over the whole paper.

 

## The strengths and weaknesses of the candidates in the treatment of individual questions

**Question 1**

**Part A**

(a) Most candidates were able to deduce the expression for the area of the minor segment correctly.

(b) This part of the question was also correctly answered by most students, mostly through subtracting the area of the minor segment from the area of the circle; a few added the areas of the major sector and triangle AOB.

(c) There were many completely correct answers to this part too. The candidates who did not get full marks here were usually those who had not reached the correct expression for the area of the major segment in (b), and those who were not able to translate the given ratio into a correct equality from which to start working.

(d) Many candidates found the correct value of $\theta$, although quite a few did not attempt this part of the question. Others did not realise a gdc approach was called for here and tried to solve the equation in (c) analytically.

**Part B**

(a) Many satisfactory answers here, with full marks being obtained by quite a high number of students. A few missed out important steps in the proof and as usual, many left out the final sentence in the proof.

(b) Again a question which required a gdc approach – many answered correctly.

**Question 2**

**Part A**

(a) Most candidates identified the situation correctly as a binomial distribution problem and found the expected number of yellow ribbons successfully. Many however seemed uncomfortable with the answer not being an integer and rounded up or offered “2 or 3” as their final answer.

(b) Correct answers from many candidates here.

(c) Most were able to find this probability also.

(d) This part of the question was not satisfactorily answered by many students – many did not attempt it and others just gave the number 2 as the answer with no working, and obtained no marks for it.

(e) Very few correct explanations here.

**Part B**

(a) Although most candidates knew that they needed to solve the equation $\int_0^k \frac{x}{1+x^2}\,dx = 1$, not so many were able to go on from here and use substitution to integrate and then find the exact value of $k$.

(b) Many students answered this correctly and efficiently by looking for the maximum value of $f(x)$ on their gdc; others took the long way round, differentiated and found the maximum value analytically.

 

(c) Correctly answered by those who realised they could use their calculators to find the required integral, although many who tried an analytical approach were also successful.

**Question 3**

**Part A**

(a) Very few candidates were not able to answer this part correctly.

(b) Although most candidates knew what was required here and many did so correctly, a disappointing number lost a mark through writing for example $l_2 = ....$

(c) Many students obtained at least a few marks for part (i) as they were able to set out the problem correctly and gave evidence that they knew what they were aiming at. Algebraic difficulties prevented many from obtaining full marks here. Most candidates found the distance between the two lines correctly, consistent with their answer to part (i).

**Part B**

(a) Although some candidates worked with the determinant of the coefficient matrix and looked for the values of k which did not make this determinant 0, many others set out on complicated row reduction methods which in very few cases were totally successful.

(b) Relatively good analysis of the two cases was seen here, although often the final conclusion for each value of k was not stated clearly enough.

**Question 4**

(a) Mostly very well answered, applying quotient rule in most cases, although a few as usual prefer to transform the function so as to be able to use product rule. The most common mistake, surprisingly, was in the differentiation of $(1-3\ln x)$ in the numerator of the second derivative.

(b) (i) The vast majority of students found the required x-coordinate correctly and on the whole also justified that it was a maximum correctly.

(ii) Although most solved $f``(x)=0$ correctly (or at least consistently from their answer to (a), but completely convincing justifications that there was a point of inflexion at this value of x were relatively rare.

(iii) Sketches were not in general very satisfactory. Many ignored the restriction on the domain, the scale was omitted in many cases and there, overall, too many untidy, poor quality graphs.

(c) Most candidates wrote the required integral for the volume correctly, but then failed to realise they could use their calculator to evaluate it. However, there were also quite a number of completely correct answers.

(d) Not so many successful answers here, although again, the correct integral was in most cases given, and most candidates knew to use integration by parts in order to solve it. Many arrived at the given answer; others made mistakes in the integration and got lost on the way. A few still think that verifying that the value obtained on the calculator for the integral coincides with the value of $\frac{1}{18}(4-\ln 3)$ is a valid strategy for this type of question.

 

**Question 5**

(a) Good answers to this part of the question from most candidates.

(b) In contrast, very few were able to do this part successfully. Many simply did not understand what was being asked; of those who did, most got as far as integrating, but forgot the constant of integration and therefore missed the last few steps of the proof and forfeited three marks.

(c) Again, very few satisfactory answers to this very straightforward deduction.

(d) Many candidates did not attempt this part of the question. Those who did usually expanded $\left(\cos\theta + i\sin\theta\right)^6$ correctly but then relatively few went on to equate imaginary parts to come to the given expression. The limit required in part (ii) was usually successfully found by those who had completed part (i).

## Recommendations and guidance for the teaching of future candidates

Emphasise the importance of clearly presented and neat work. Sometimes it becomes practically impossible to award any method marks at all because it is, to all intents and purposes, hopelessly impossible to understand candidates´ reasoning.

# Higher level paper three

## Component grade boundaries

| **Grade**: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range**: | 0 - 8 | 9 - 17 | 18 - 25 | 26 - 32 | 33 - 38 | 39 - 45 | 46 - 60 |

## General comments

This report was unavailable at the time of publication. It will be added shortly. Please accept our apologies.

## The areas of the programme and examination that appeared difficult for the candidates

## The areas of the programme and examination in which candidates appeared well prepared

## The strengths and weaknesses of the candidates in the treatment of individual questions

## Recommendations and guidance for the teaching of future candidates

 